

Algorithms for Hiring and Outsourcing in the Online Labor Market^{*†}

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ABSTRACT

Although freelancing work has grown substantially in recent years, in part facilitated by a number of online labor marketplaces, traditional forms of “in-sourcing” work continue being the dominant form of employment. This means that, at least for the time being, freelancing and salaried employment will continue to co-exist. In this paper, we provide algorithms for outsourcing and hiring workers in a general setting, where workers form a team and contribute different skills to perform a task. We call this model *team formation with outsourcing*. In our model, tasks arrive in an online fashion: neither the number nor the composition of the tasks are known a-priori. At any point in time, there is a team of hired workers who receive a fixed salary independently of the work they perform. This team is dynamic: new members can be hired and existing members can be fired, at some cost. Additionally, some parts of the arriving tasks can be outsourced and thus completed by non-team members, at a premium. Our contribution is an efficient online cost-minimizing algorithm for hiring and firing team members and outsourcing tasks. We present theoretical bounds obtained using a primal-dual scheme proving that our algorithms have logarithmic competitive approximation ratio. We complement these results with experiments using semi-synthetic datasets based on actual task requirements and worker skills from three large online labor marketplaces.

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^{*}Because of space limitations, some details and proofs have been omitted, they will appear in the full version of this work.

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1 INTRODUCTION

Self-employment is an increasing trend; for instance, between 10% and 20% of workers in developed countries are self-employed [24]. This phenomenon can be partially attributed to business downsizing and employee dissatisfaction, as well as to the existence of online labor markets (e.g., *Guru.com*, *Freelancer.com*). This trend has enabled freelancers to work remotely on specialized tasks, and prompted researchers and practitioners to explore the benefits of outsourcing and *crowdsourcing* [14, 15, 17, 22, 25, 28].

Although crowdsourcing adoption was driven, at least in part, by the assumption that problems can be decomposed into parts that can be addressed separately by independent workers, crowdsourcing results can be improved by allowing some degree of collaboration among them [20, 26]. The idea of combining collaboration with crowdsourcing has led to research on *team formation* [2–4, 10–12, 16, 18, 19, 21, 27], in which a common thread is the need for complementary skills, and problem settings differ in aspects such as objectives (e.g., load balancing and/or compatibility), constraints (e.g., worker capacity), and algorithmic set up (online or offline).

Overview of problem setting and assumptions. We consider tasks that arrive in an online fashion and must be completed by assigning them to one or more workers, who jointly cover the skills required for each task. At any point in time, there is a team of hired workers who are paid a salary, independently of the work they perform. This team is dynamic: new members can be hired and existing members can be fired. Hiring and firing workers is expensive, which is why companies routinely keep on the payroll skilled workers even if they are temporarily idle; however, they also seek to maintain “benching” to a minimum [29]. Outsourcing provides additional flexibility as some parts of the incoming tasks can be completed by non-team members who are outsourced. In practice, outsourcing involves additional costs such as searching, contracting, communicating with, and managing an expert or specialist external to a company [6].

Deciding when to hire, fire, and outsource workers is a difficult online problem with parameters that depend on job market conditions and employment regulations. Intuitively: (1) if the cost of hiring or firing workers is too high, outsourcing becomes preferable to hiring; (2) if the cost of outsourcing work relative to salaries of hired workers is too high, hiring becomes preferable to outsourcing; and (3) if the workload consists of many repetitions of similar tasks, hiring becomes preferable to outsourcing.

In this paper, we formulate this as an online cost minimization problem, which we call Team Formation with Outsourcing (TFO). We formally define this problem in Section 2 and solve it in Sections 3 and 4. Despite this being a model and hence not capturing

every aspect of employment decisions in a company, we show how it brings formalism to the intuitions we have outlined, helps understand under which circumstances a combination of hiring and outsourcing can be cost effective, and motivates experimentation on semi-synthetic data allowing us to cover a broad range of cases, as we show in Section 5.

Algorithmic techniques. To the best of our knowledge, we are the first to consider this problem and study some of its variants. Our problem turns out to be an original generalization of *online set cover* and *online ski rental*, two of the most paradigmatic online problems. In fact TFO has elements that make it more complex; to solve it, an algorithm has to address its various characteristics: (1) it is also online, so decisions should be taken with limited information on the input, but at each step, an entirely new instance of the set-cover problem needs to be solved by using hired and outsourced workers; (2) hired and outsourced workers collaborate with each other, and this needs to be taken into account; and (3) workers can be hired, fired, hired again, and so on, so one has to keep track of their status at every point in time.

Several natural approaches inspired by online algorithms for the problems we mentioned previously, fail to provide solutions with theoretical guarantees. Therefore, we consider an approach introduced in the last years for studying complex online problems, the *online primal-dual* scheme [7]. The idea is to create a sequence of integer programs to model the online problem by incrementally introducing variables and constraints. We then consider their linear relaxations and their duals to design an online algorithm and we analyze it by comparing the costs of the primal and the dual programs as they evolve over time with the arrival of new tasks. This is a powerful approach, which has so far been applied with success to several classical online problems: packing and covering problems, ski-rental, weighted caching, k-server among others [5]. We refer to [5, 7] for a survey of the applications of the online primal-dual method.

Our analysis results in polynomial-time algorithms that have logarithmic competitive approximation ratios. This means that despite the fact that our algorithms work in an online fashion and they do not have any knowledge of the number and the composition of future tasks, we can guarantee that the cost they will incur will be, *at every time instance*, only a logarithmic factor worse than the cost incurred by an optimal algorithm that knows the set of requests a priori.

Contributions. The key contributions of our work are:

- We formalize TFO: the problem of designing an online cost-minimizing algorithm for hiring, firing and outsourcing.
- We design efficient and effective approximation algorithms for TFO using an online primal-dual scheme, and provide approximation guarantees on their performance.
- We experiment on semi-synthetic data based on actual task requirements and worker skills from three large online labor marketplaces, testing algorithms under a broad range of conditions.
- We provide experimental evidence of the quality of the performance of online primal-dual algorithms for a complex real-world problem. Prior work has performed theoretical analysis mostly for classical or practically motivated online problems [8, 9]. To

the best of our knowledge, the empirical validation was previously addressed only for the Adwords matching problem [13]. We demonstrate that such approaches, even though they are based on heavy theoretical machinery, can be easily implemented and are efficient in practice.

2 PRELIMINARIES

In this section, we formally describe our setting and problem, and provide some necessary background.

2.1 Notation and Setting

Skills. We consider a set S of skills with $|S| = m$. Skills can be any kind of qualification a worker can have or a task may require, such as *video editing*, *technical writing*, or *project management*.

Tasks. We consider a set of T^* tasks (or jobs), $\mathcal{J} = \{J^t; t = 1, 2, \dots, T^*\}$, which arrive one-by-one in a streaming fashion; J^t is the t th task that arrives. Each task $J \in \mathcal{J}$ requires a set of skills from S , therefore, $J \subseteq S$. We use J^t to refer to both the task and the skills that it requires.

Workers. Throughout we assume that we have a set \mathcal{W} of n workers: $\mathcal{W} = \{W^r; r = 1, \dots, n\}$. Every worker r possesses a set of skills ($W^r \subseteq S$), and P_ℓ denotes the subset of workers possessing a given skill ℓ : $P_\ell = \{r; \ell \in W^r\}$. Similarly to the tasks, we use W^r to denote both the worker and his/her skills.

We partition the set of available workers \mathcal{W} into the set of workers who are *hired* at time t , denoted by \mathcal{H}^t , and the set of workers who are *not hired*, denoted by \mathcal{F}^t (we sometimes refer to these workers as *freelancers*, and they can be *outsourced* for J^t), so that $\mathcal{H}^t \cap \mathcal{F}^t = \emptyset$ and $\mathcal{W} = \mathcal{H}^t \cup \mathcal{F}^t$.

Coverage of tasks. Whenever task $J^t \subseteq S$ arrives, an algorithm has to assign one or more workers to it, i.e., a *team*. We say that J^t can be *completed* or *covered* by a team $Q \subseteq \mathcal{W}$ if for every skill required by J^t , there exists at least one worker in Q who possesses this skill: $J^t \subseteq \cup_{W \in Q} W$. We assume that for every skill in the incoming task there is at least one worker possessing that skill, so all tasks can be covered.

Costs. Every worker W^r potentially can charge the following non-negative, worker-specific fees: (1) an *outsourcing fee* λ_r , (2) a *hiring fee* C_r , and (3) a *salary* σ_r . Outsourcing fees λ_r denote the payment required by a (non-hired) worker when a task is outsourced to him/her. Note that λ_r depends on the worker but does not depend on the task. Hiring fees C_r reflect all expenses associated to hiring and firing a worker, such as signup bonuses and severance payments. Given that any algorithm commits to pay the firing costs the moment in which it hires a worker, we follow a standard methodology used in online algorithms for caching [7] and account for both hiring and firing costs when the worker is hired. Once a worker r is hired, s/he is paid a recurring salary σ_r , which recurs for every step t that the worker is hired. The above notation is summarized on Table 1.

Assumptions. To avoid making the model overly complicated, we assume that the salary periods are defined by the arriving tasks, this is, there is one task per salary period, and task completion takes one salary period. A further assumption will be that $\sigma_r < \lambda_r$, as in practice requesting a single task from an external worker involves

Table 1: Notation

S	Set of skills, size m
\mathcal{J}	Set of tasks, size T^*
T	Number of tasks till current time
J^t	The t 'th task arriving
	$J_\ell^t = 1$ if task t requires skill ℓ , 0 otherwise
\mathcal{W}	Set of workers, size n .
	$W_\ell^r = 1$ if worker r possess skill ℓ , 0 otherwise
P_ℓ	Subset of workers possessing skill ℓ
C_r	Hiring fee, paid when worker r is hired
λ_r	Outsourcing fee, paid every time r performs a task
σ_r	Salary paid to a hired worker r

extra costs [6], which are reduced when the worker is hired (or when an outsourcing arrangement for an external group of workers to perform a specific recurring task is done, which is different from the individual outsourcing we discuss here). Finally, we assume $\lambda_r < C_r + \sigma_r$, because otherwise workers would be hired and fired for every task.

2.2 Problem Definition

We now define the problem that we study:

PROBLEM 1 (TEAM FORMATION WITH OUTSOURCING – TFO). *There exists a set of skills S . We have a pool of workers \mathcal{W} , where each worker $W^r \in \mathcal{W}$ is characterized by a subset of skills $W^r \subseteq S$, an outsourcing cost $\lambda_r \in \mathbb{R}_{\geq 0}$, a hiring cost $C_r \in \mathbb{R}_{\geq 0}$, and a salary cost $\sigma_r \in \mathbb{R}_{\geq 0}$. Given a set of tasks $\mathcal{J} = \{J^1, J^2, \dots, J^{T^*}\}$, with $J^t \subseteq S$, which arrive in a streaming fashion, the goal is to design an algorithm that, when task J^t arrives, decides which workers to hire (paying cost $C_r + \sigma_r$), keep hired (paying cost σ_r), and outsource (paying cost λ_r), such that all the tasks are covered by the workers who are hired or outsourced and the total cost paid over all the tasks is minimized.*

TFO is an online problem: \mathcal{J} is revealed one task at a time. Our goal is to guarantee that for *any* input stream \mathcal{J} the total cost of our online algorithm, $ALG(\mathcal{J})$, is at most a small factor greater than the total cost of the optimal (offline) algorithm that knows \mathcal{J} in advance, $OPT(\mathcal{J})$. This factor, $\max_{\mathcal{J}} ALG(\mathcal{J})/OPT(\mathcal{J})$, is called the *competitive ratio* of the algorithm.

We solve the TFO problem in Section 4. Because neither the algorithm nor its analysis are trivial, we introduce them gradually by first solving a simplified version of TFO, which we describe and solve in Section 3.

2.3 Background Problems

Two special cases of TFO are SETCOVER and SKIRENTAL.

SETCOVER: the single-task, multiple-skill case. The set cover problem is an instance of our problem when there is a single task $J \subseteq S$ and for each worker W^r , $C_r = \infty$. Then, as soon as the task J arrives, the algorithm needs to cover all skills in J by selecting a set of workers $Q \subseteq \mathcal{W}$ such that Q covers J and $\sum_{r \in Q} \lambda_r$ is minimized. In this case, our problem can be solved using the greedy algorithm for the set-cover problem (see [30, Chapter 2]).

SKIRENTAL: the single-skill, single-worker case. The ski rental problem is an instance of our problem when the sequence of

tasks \mathcal{J} consists of a repetition of the same single-skill task J and the workforce \mathcal{W} consists of a single worker W^r who possesses the same one skill, and has $\sigma_r = 0$ and some C_r, λ_r . In this ski-rental version of our problem [23], the question is the following: without knowledge of the total number of tasks that will arrive, when should worker W be hired so that the total cost paid to him/her in outsourcing plus hiring fees is minimized?

A well-known algorithm for this problem is the following: for every instance of J^t that arrives outsource J^t to worker W^r as long as: $\sum_{t'=1}^t \lambda_r < C_r$. Then, hire the worker when $\sum_{t'=1}^t \lambda_r \geq C_r$. The above algorithm achieves a competitive ratio of 2.

3 THE LUMP SUM PROBLEM

First, we solve a simplified version of the TFO problem, where for every worker W^r the salary is equal to 0 ($\sigma_r = 0$). In this version of the problem, which we call LUMP SUM, a hired worker W^r is paid a lump sum of C_r the moment s/he is hired and this amount is assumed to cover all future work done by the worker. Instead, when a worker W^r is outsourced, a payment of λ_r is done every time s/he performs a task.

3.1 The LumpSum-Heuristic Algorithm

A natural algorithm for solving the LUMP SUM problem combines ideas from SETCOVER and SKIRENTAL as follows: first, it starts with no worker being hired and each worker W^r is associated with a variable δ_r initially set to 0.

For any $T \in \{1, \dots, T^*\}$, when task J^T arrives, the algorithm proceeds as follows: first, it identifies $J_{\mathcal{F}}^T$ to be the set of skills of J^T that cannot be covered by already-hired workers. Then, it covers the skills in $J_{\mathcal{F}}^T$ using the greedy algorithm for SETCOVER. This way it finds $Q^T \subseteq \mathcal{W}$ such that $\sum_{W^r \in Q^T} \lambda_r$ is minimized. Finally, for each worker $W^r \in Q^T$, it updates $\delta_r \leftarrow \delta_r + \lambda_r$. Worker W^r is hired when $\delta_r \geq C_r$. Clearly, since there are no salaries there is no motivation to fire a worker once s/he is hired.

LumpSum-Heuristic has arbitrarily bad competitive ratio. Although our experiments (Section 5) demonstrate that the above algorithm, which we call LumpSum-Heuristic, performs quite well in many practical cases, we can show that its competitive ratio can be arbitrarily bad. For this, consider an example where $\mathcal{W} = \{W^1, W^2\}$ and both workers have the same skill: $W^1 = W^2 = \{\ell\}$. Further assume that $\lambda_1 = 1$, $\lambda_2 = 1 + \epsilon$ and $C_1 = M$, $C_2 = 2$, where M is a large value and ϵ a small one. For a sequence of tasks $J^1 = J^2 = \dots = J^{T^*} = \{\ell\}$, it is clear that LumpSum-Heuristic will always outsource to W^1 until hiring him/her and will incur worst-case cost $2M$, whereas the optimal algorithm pays just $C_2 = 2$.

3.2 A Primal–Dual Algorithm

The above discussion illustrates that to obtain an algorithm with bounded competitive ratio, we need to take into account both the outsourcing and hiring costs of all workers. To do so, we deploy an online primal–dual scheme, which drives our algorithm design.

The integer and linear programs. The first step of the primal–dual approach, is to define an integer formulation for the problem, for each step $T \in \{1, \dots, T^*\}$. We assume that the current task is the T th task and we use the following variables:

- $x_r = 1$ if worker W^r is hired when task J^T arrives; otherwise $x_r = 0$.
- $f_{rt} = 1$ if worker W^r is outsourced for performing task J^t ; otherwise $f_{rt} = 0$.

Using this notation, LUMPSUM can be formulated as follows:

Linear program for LUMPSUM:

$$\min \sum_{r=1}^n \left(C_r x_r + \lambda_r \sum_{t=1}^T f_{rt} \right) \quad \text{subject to:}$$

$$\forall t = 1, \dots, T, \ell \in J^t: \quad \sum_{W^r \in P_\ell} (x_r + f_{rt}) \geq 1 \quad (1)$$

$$\forall t = 1, \dots, T, r = 1, \dots, n: \quad x_r, f_{rt} \geq 0$$

The above, in addition to the integrality constraints $x_r, f_{rt} \in \mathbb{N}$, form the integer program for LUMPSUM. In this formulation, the objective function sums over all workers the hiring costs (paid if the corresponding worker has been hired by time t) and the outsourcing cost for the tasks for which the worker has been outsourced. This is the total cost of the solution until the current task J^T . Note that in this formulation of the problem there is no motivation for a worker who is hired to be fired. Therefore, once x_r is set to 1, it does not change its value to become 0 again.

The first constraint (1) in the above program is the covering constraint: it simply enforces that for every skill required for each task, there exists a hired or outsourced worker who has this skill. This guarantees that the team selected for each task J^t covers *all* the required skills. The nonnegativity and the integrality constraints, ensure that the solutions that we obtain from the integer-program formulation can be transformed to a solution to our problem: eventually, every variable will take the value 0 or 1.¹

To apply the online primal-dual approach, we first consider the linear relaxation of the integer program, which simply drops the integrality constraints $x_r, f_{rt} \in \mathbb{N}$. In a solution to this linear program (LP) each variable takes values in $[0, 1]$. Given this LP, we can write its dual as follows:

The dual of the linear program for LUMPSUM:

$$\max \sum_{t=1}^T \sum_{\ell \in J^t} u_{\ell t} \quad \text{subject to:}$$

$$\forall r = 1, \dots, n: \quad \sum_{t=1}^T \sum_{\ell \in J^t \cap W^r} u_{\ell t} \leq C_r \quad (2)$$

$$\forall t = 1, \dots, T, r = 1, \dots, n: \quad \sum_{\ell \in J^t \cap W^r} u_{\ell t} \leq \lambda_r \quad (3)$$

$$\forall t = 1, \dots, T, \ell \in J^t: \quad u_{\ell t} \geq 0$$

Note that at every time $t \in \{1, \dots, T\}$ we have such a pair of primal-dual formulations. We are now going to use these two formulations for designing and analyzing our algorithm.

¹A solution in which some variables take values greater than 1, can be transformed to another feasible solution with lower cost by setting these variables to 1.

The LumpSum algorithm: Next, we present the LumpSum algorithm, which is designed and analyzed using the primal and the dual linear programs. We assume that task J^T , $T \in \{1, \dots, T^*\}$, has just arrived and the algorithm must act before task J^{T+1} arrives (or the stream finishes if $T = T^*$).

All the variables used in our algorithm are initialized to 0 before the arrival of the first task. When task J^T arrives the following steps are done:

1. Let \mathcal{F}^T and \mathcal{H}^T represent the workers who are *not hired* and *hired*, respectively, at the time that J^T arrives. Clearly, when the first task arrives ($T = 1$), $\mathcal{F}^T = \mathcal{W}$ and $\mathcal{H}^T = \emptyset$. For $T > 1$, the values of \mathcal{H}^T and \mathcal{F}^T are updated in the last step (step 10) of the previous round.
2. Let $J_{\mathcal{H}}^T = J^T \cap \cup_{W^r \in \mathcal{H}^T} W^r$ be the skills from J^T that are covered by already-hired workers and $J_{\mathcal{F}}^T = J^T \setminus J_{\mathcal{H}}^T$.
3. For every skill $\ell \in J_{\mathcal{F}}^T$ let $P_\ell^{\mathcal{F}} = P_\ell \cap \mathcal{F}^T$ be the set of workers in \mathcal{F}^T such that every worker in $P_\ell^{\mathcal{F}}$ has skill ℓ . Also let $P_T^{\mathcal{F}} = \cup_{\ell \in J_{\mathcal{F}}^T} P_\ell^{\mathcal{F}}$ be the set of unhired workers who possess at least one skill that is required and not covered by already-hired workers.
4. **for each** $W^r \in P_T^{\mathcal{F}}$: set $\tilde{x}_r \leftarrow \tilde{x}_r$.
5. **for each** skill $\ell \in J_{\mathcal{F}}^T$:
while $\sum_{W^r \in P_\ell} (\tilde{x}_r + \tilde{f}_{rT}) < 1$:
for each $W^r \in P_\ell$: $\tilde{x}_r \leftarrow \tilde{x}_r \left(1 + \frac{1}{C_r}\right) + \frac{1}{nC_r}$
for each $W^r \in P_\ell$: $\tilde{f}_{rT} \leftarrow \tilde{f}_{rT} \left(1 + \frac{1}{\lambda_r}\right) + \frac{1}{n\lambda_r}$
6. **for each** $W^r \in P_T^{\mathcal{F}}$: set $\Delta \tilde{x}_r \leftarrow \tilde{x}_r - \tilde{x}'_r$.
7. Set $\mathcal{H}' \leftarrow \emptyset$.
8. **repeat** ρ_1 times:
for each $W^r \in P_T^{\mathcal{F}}$
with probability $\Delta \tilde{x}_r$:
hire worker W^r (set $x_r \leftarrow 1, \mathcal{H}' \leftarrow \mathcal{H}' \cup \{r\}$)
with probability \tilde{f}_{rT} :
outsource worker W^r (set $f_{rT} \leftarrow 1$)
9. **for each** skill $\ell \in J_{\mathcal{F}}^T$:
if skill ℓ is not covered:
hire worker $W^r \in P_\ell^{\mathcal{F}}$ with minimum cost C_r
(set $x_r \leftarrow 1, \mathcal{H}' \leftarrow \mathcal{H}' \cup \{r\}$)
10. $\mathcal{H}^{T+1} \leftarrow \mathcal{H}^T \cup \mathcal{H}'$, $\mathcal{F}^{T+1} \leftarrow \mathcal{W} \setminus \mathcal{H}^{T+1}$.

For $T = 1$, the LumpSum starts with no worker being hired. Intuitively, as tasks arrive, the algorithm tries to gauge two quantities: (1) the usefulness of every worker for the task at hand J^T and (2) the overall usefulness of every worker for tasks J^1, \dots, J^T . This is done in step 5, via variables \tilde{f}_{rT} (for (1)) and \tilde{x}_r (for (2)). In particular, the more useful W^r proves over time, the larger the value \tilde{x}_r . Subsequently, in step 8 every worker is outsourced or hired based on the increase in the values of \tilde{f}_{rT} and \tilde{x}_r observed in step 5. Finally, for every skill that remains uncovered after step 8 (which is randomized), LumpSum hires worker W^r with the minimum C_r that covers the skill. Note that the increase of the variables $u_{\ell T}$ in step 5 is not required for solving the LUMPSUM, but it is used in our analysis and thus we leave it in the description above.

Our analysis requires to set the value of ρ_1 in step 8 to $\rho_1 = \ln m + \ln C^*$, where $C^* = \max_{W^r \in \mathcal{W}} C_r$.

Although one may think that an additive update of variables in step 5 would seem more natural, such an update would introduce a $\Theta(m)$ factor in the competitive ratio. On the other hand, the multiplicative update that we adopt, has the property that the more a worker W^r is required over time the higher the increase of the corresponding variable \tilde{x}_r . This fact, leads us to Theorem 3.1 below.

Analysis. We have the following result for LumpSum.

THEOREM 3.1. *LumpSum is an $O(\log n(\log m + \log C^*))$ -competitive algorithm for the LUMP SUM problem, where $C^* = \max_{W^r \in \mathcal{W}} C_r$.*

Running time. The running time of LumpSum per task is dominated by the execution of steps 5 and 8. For step 5, using binary search, the algorithm can determine in $O(\log C^*)$ steps the minimum increase of \tilde{x}_r and \tilde{f}_{rT} that makes false the condition of the while loop for at least one uncovered skill ℓ . Therefore, the running time of step 5 is $O(|J^T| n \log C^*)$. Step 8, using a hash table to store hired workers, can be executed in expected time $O(\rho_1 n) = O(n(\log m + \log C^*))$. Therefore, the expected time required for processing task J^T is $O(n(\log C^* |J^T| + \log m))$.

4 THE TFO PROBLEM

In this section, we provide an algorithm for the general version of TFO (Problem 1). In contrast with LUMP SUM, now after hiring a worker we must pay a salary $\sigma_r \geq 0$, complicating the problem significantly as it may now be cost-effective to fire workers.

The integer and linear programs for TFO. Given that workers can be hired, then fired and potentially hired again, and so on, we introduce in this new LP the notion of intervals. These intervals are used to model periods in which workers are hired $\mathcal{I} = \{\{t_a, t_b\} \mid t_a, t_b \in \mathbb{N}, t_a \leq t_b\}$. Intuitively, an interval is a subset of time steps during which an algorithm decides to hire a given worker. The new LP, (omitted) uses the following variables:

- $x(r, I)$ with $I \in \mathcal{I}$: $x(r, I) = 1$ if worker W^r is hired during the entire interval I ; otherwise $x(r, I) = 0$.
- f_{rt} : $f_{rt} = 1$ iff worker W^r is outsourced for performing J^t .

It turns out that it is hard to design an approximation algorithm with proven guarantees using this program, mostly because it is hard to keep track of the costs being paid for every worker when the intervals of him/her being hired, outsourced, or idle are of variable length. Therefore, we resort to a different overall *strategy*: First, we define the ALT-TFO problem, in which the solutions are restricted such that every worker is hired for fixed-length (worker-specific) intervals (Section 4.1). Then, we design an algorithm for ALT-TFO with good competitive ratio (Section 4.2). Finally, we prove that a solution to ALT-TFO can be transformed to a solution for TFO, and that any solution of TFO can be transformed to a feasible solution of ALT-TFO that is a factor of at most 3 times higher (Section 4.3), obtaining an approximation algorithm for TFO.

4.1 The ALT-TFO Problem

The difference between ALT-TFO and TFO is that we restrict the solutions of the former to have a specific structure; whenever worker W^r is hired s/he is then fired after $\eta_r \triangleq \lceil C_r / \sigma_r \rceil$ time units— independently of whether s/he is used or not in tasks within these η_r time units.

In this case, every worker W^r is associated with a new hiring cost \widehat{C}_r , which is the summation of his/her original hiring cost C_r plus the salaries paid to him/her for the η_r time units he is hired. Thus, the total hiring cost and salary for an entire interval is $C_r + \eta_r \cdot \sigma_r \leq C_r + \left(\frac{C_r}{\sigma_r} + 1\right) \cdot \sigma_r \leq 3C_r$. We will use $\widehat{C}_r \triangleq 3 \cdot C_r$.

We can now write the LP for ALT-TFO. In addition to the notation we discussed in the previous paragraph, we use $I^t \in \mathcal{I}$ to denote the interval that starts at time t . Worker W^r has $x(r, I) = 1$ iff s/he is hired during the *entire* interval I . All intervals I for which $x(r, I) = 1$ are of fixed length η_r .

Linear program for ALT-TFO:

$$\min \sum_{r=1}^n \left[\sum_{I \in \mathcal{I}} \widehat{C}_r x(r, I) + \sum_{t=1}^T \lambda_r f_{rt} \right] \quad \text{subject to:}$$

$$\forall t = 1 \dots T, \ell \in J^t: \quad \sum_{W^r \in P_\ell} \left(f_{rt} + \sum_{I \in \mathcal{I}: t \in I} x(r, I) \right) \geq 1 \quad (4)$$

$$\forall t = 1 \dots T, r = 1 \dots n, I \in \mathcal{I}: \quad x(r, I), f_{rt} \geq 0$$

4.2 Solving the ALT-TFO Problem

In this section, we design and analyze an algorithm for the ALT-TFO problem. The similarity between the LPs for ALT-TFO and LUMP SUM (Section 3) translates into a similarity in the algorithms (and their analysis) of the two problems. The key difference now is that we need to take care of the firings.

Our algorithm for ALT-TFO differs from the algorithm for LUMP SUM in steps 1, 5, 8, and 9, which are changed as follows:

- 1'. Let \mathcal{F}^T and \mathcal{H}^T represent the workers who are not hired and hired, respectively, at the time that J^T arrives. Clearly, when the first task arrives ($T = 1$), then $\mathcal{F}^T = \mathcal{W}$ and $\mathcal{H}^T = \emptyset$. For $T > 1$, the values of \mathcal{H}^T and \mathcal{F}^T are updated in the last step (step 10) of the previous round and then we remove workers whose hiring interval finished in the previous step:

$$\mathcal{F}' \leftarrow \{r \in \mathcal{W}; x(r, I^{T-\eta_r}) = 1\}$$

$$\mathcal{H}^T \leftarrow \mathcal{H}^T \setminus \mathcal{F}', \quad \mathcal{F}^T \leftarrow \mathcal{W} \setminus \mathcal{H}^T$$
for each $W^r \in \mathcal{F}'$: **set** $\tilde{x}_r \leftarrow 0$
- 5'. **for each** skill $\ell \in J_{\mathcal{F}^T}^T$:
 while $\sum_{r \in P_\ell} (\tilde{x}_r + \tilde{f}_{rT}) < 1$:
 for each $r \in P_\ell$: $\tilde{x}_r \leftarrow \tilde{x}_r \left(1 + \frac{1}{C_r}\right) + \frac{1}{n\widehat{C}_r}$
for each $r \in P_\ell$: $\tilde{f}_{rT} \leftarrow \tilde{f}_{rT} \left(1 + \frac{1}{\lambda_r}\right) + \frac{1}{n\lambda_r}$
- 8'. **repeat** $\rho_2(T)$ times:
 for each $r \in P_{\mathcal{F}^T}^{\mathcal{F}}$
 with probability $\Delta \tilde{x}_r$:
 hire worker W^r (set $x(r, I^T) \leftarrow 1, \mathcal{H}' \leftarrow \mathcal{H}' \cup \{r\}$)
 with probability \tilde{f}_{rT} :
 outsource worker W^r (set $f_{rT} \leftarrow 1$)
- 9'. **for each** skill $\ell \in J_{\mathcal{F}^T}^T$:
 if skill ℓ is not covered:
 outsource worker $W^r, r \in P_\ell^{\mathcal{F}}$, with minimum cost λ_r
 (set $f_{rT} \leftarrow 1$)

Our analysis requires to set $\rho_2(T) = \ln m + \ln \lambda^* + 2 \ln T$, where $\lambda^* = \max_{W^r \in \mathcal{W}} \lambda_r$.

Analysis of Alt-TFO. Algorithm Alt-TFO gives a solution with proven theoretical guarantees for ALT-TFO. As before, the multiplicative update is needed to obtain this competitive ratio. We have the following theorem (proof omitted due to space constraints).

THEOREM 4.1. *Alt-TFO is an $O(\log n(\log m + \log \lambda^* + \log T^*))$ -competitive algorithm for the ALT-TFO problem.*

4.3 Solving TFO Using ALT-TFO

Note that any solution output by Alt-TFO can be transformed into a feasible solution to the original TFO problem by setting $g_{rt} \leftarrow 1$ for each $r, t \in I$ for which $x(r, I) = 1$, and $g_{rt} \leftarrow 0$ otherwise. We call the algorithm that runs Alt-TFO and subsequently does this transformation its final step, the TFO algorithm.

The question is whether TFO provides a solution with bounded competitive ratio for the TFO problem. We answer this question affirmatively by showing (1) that the solution of TFO for the TFO problem is feasible and has a cost bounded by the cost of Alt-TFO for the ALT-TFO problem, and (2) that any solution for the TFO problem can be turned into a feasible solution to the ALT-TFO problem at the expense of a small loss in the approximation factor. These two suffice to prove that the solution produced by TFO is a good solution for the TFO problem. We have the following result (proof omitted due to space constraints):

THEOREM 4.2. *TFO is an $O(\log n(\log m + \log \lambda^* + \log T^*))$ -competitive algorithm for the TFO problem.*

Running time. Similarly to Section 3, the expected time required to process task J^T is $O(n(\log C^* |J^T| + \log m + \log T))$.

Lower bound. Note that there is little hope for significant improvement of our theoretical results. In particular, Alon et al. [1] have proven a lower bound of $\Omega\left(\frac{\log n \log m}{\log \log n + \log \log m}\right)$ on the competitiveness of any deterministic algorithm for the *unweighted online set cover problem*. The unweighted online set cover problem, is a special case of TFO (and of LUMP SUM) where for each worker W^r we have $C_r = \lambda_r = 1$, $\sigma_r = 0$, and for each task J^T we have $J^{T-1} \cup \{\ell\}$, for some skill $\ell \in S \setminus J^{T-1}$ (with $J^0 = \emptyset$).

4.4 The TFO-Heuristic

Similarly to LUMP SUM, we also consider the heuristic TFO-Heuristic, which is a generalization of LumpSum-Heuristic, for general values of σ_r . Specifically, the difference is that worker W^r is hired when $\delta_r \geq C_r + \eta_r \cdot \sigma_r$, and is fired after η_r tasks (see Sections 3.1 and 4.1 for definitions of δ_r and η_r). Note that theoretically TFO-Heuristic may perform arbitrarily bad: the example of Section 3.1 holds for TFO-Heuristic for small σ_r . Yet, in Section 5 we observe that even though it does not offer the theoretical guarantees of TFO, it performs well in practice.

4.5 The TFO-Adaptive algorithm

As we will see in Section 5, although TFO gives theoretical guarantees for the worst-case performance, in practice some of our other algorithms for the TFO problem may perform better under some input parameters. Given the low running time of all our solution

Table 2: Characteristics of the three source datasets used to generate workloads for our experiments. Numbers in italics correspond to tasks generated for the Upwork dataset, as explained in Section 5.1.

Dataset	UpWork	Freelancer	Guru
Skills (m)	2,335	175	1,639
Workers (n)	18,000	1,211	6,119
Tasks (T)	<i>50,000</i>	992	3,194
... distinct	<i>50,000</i>	600	2,939
... avg. similarity (Jaccard)	<i>0.095</i>	0.045	0.018
Average Skills/worker	6.29	1.45	13.07
Average Skills/task	<i>41.88</i>	2.86	5.24

approaches to TFO, we implemented the TFO-Adaptive algorithm. This algorithm runs in parallel all the presented methods for solving the TFO problem (TFO, TFO-Heuristic, Always-Outsource and Always-Hire), and selects at each time the current minimum-cost algorithm to apply to solve the current task, switching between algorithms when it is advantageous. The asymptotic worst-case results hold for the TFO-Adaptive algorithm as well. Furthermore, our experiments (see Section 5) show that it is beneficial to change the hiring policy even if we pay switching costs.

5 EXPERIMENTS

Our experiments seek to compare the total cost that would be incurred by companies using different algorithms to assign workers to a stream of incoming tasks. We use synthetic datasets representing possible workloads, built using actual task requirements and worker skills from three large online marketplaces. Synthetic data, while having the limitation of not reflecting the particular conditions of a specific company, allows us to evaluate the effectiveness of our algorithms under a broad range of conditions. Section 5.1 introduces our datasets, Section 5.2 presents results on the LUMP SUM problem, and Section 5.3 on the TFO problem.

5.1 Datasets

We start by introducing our datasets and discussing our choice of cost parameters for experimentation.

Source datasets. To create a large pool of tasks from which to sample workloads, we use datasets obtained from three large online marketplaces for outsourcing: UpWork, Freelancer and Guru (the authors are not associated with any of these services). All three are in the top-30 of traffic in their category (“consulting marketplaces”) according to data from Alexa (Feb. 2018), indeed, Freelancer and Guru are respectively number 1 and number 3. General statistics of these datasets are shown on Table 2.

Worker skills. The input data that we obtained contain anonymized profiles for people registered as freelancers in these marketplaces. These include their self-declared sets of skills, as well as the average rate that they charge for their services. There is a large variation in the number of skills per worker among datasets, as can be seen in Table 2. Data have been cleaned to remove skills that were not possessed by any worker and skills that were never required by any task. The numbers in Table 2 refer to the clean datasets.

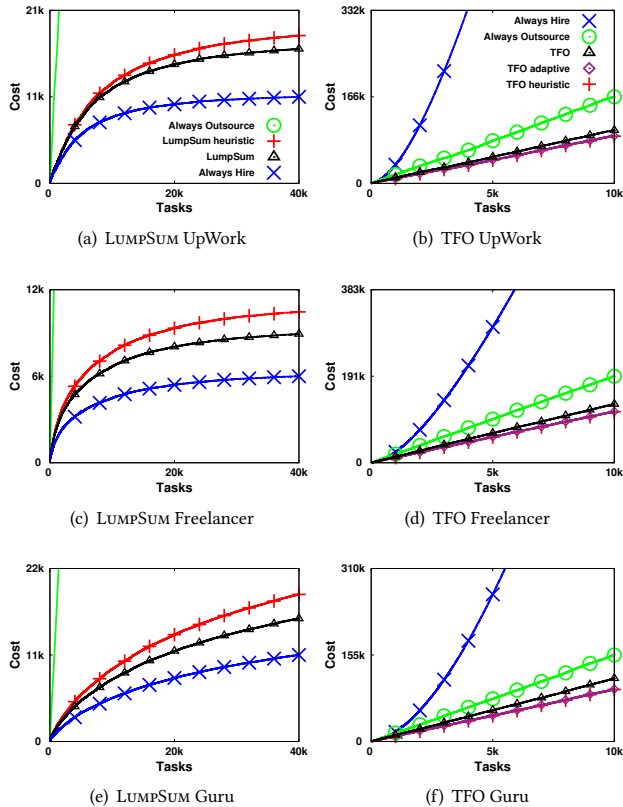


Figure 1: Experimental comparison of algorithms showing total cost due to outsourcing, hiring, and paying salaries as a function of the number of tasks in the input, averaged over 100 workloads generated with $p = 100$. Left: Algorithms for problem LUMP SUM. As expected, Always-Hire has the smallest cost if the number of tasks is large, however an online algorithm does not know the number of tasks. Our online algorithm and its heuristic version (LumpSum-Heuristic) show a cost that does not exceed twice of that of Always-Hire. In contrast, Always-Outsource has cost proportional to the number of tasks. Parameters $C_r = 4\lambda_r$ and $T = 40K$. Right: Algorithms for problem TFO. Our online algorithm, its heuristic version (TFO-Heuristic) and the TFO-Adaptive have smaller cost than Always-Outsource and Always-Hire. The latter diverges rapidly due to salary costs. Parameters $C_r = 4\lambda_r$, $\sigma_r = 0.1\lambda_r$ and $T = 10K$.

Tasks. For both Freelancer and Guru we have access to a large sample of tasks commissioned by buyers in the marketplace; they are included as tasks on Table 2. They correspond to actual tasks brought to these marketplaces by actual users. These samples are anonymized: we do not know the name of the company commissioning them, and there are no timestamps in this data. In the case of Upwork, we generate synthetic tasks following a data-generation procedure used in previous work [4]: we remove a small number of workers (10%), who are excluded from the pool of workers in the

dataset, and then repeatedly sample subsets of them to create tasks, by interpreting the union of their skills as task requirements.

Workloads. Marketplaces for online work cover a broad range of tasks from graphic design and web development to accounting, administrative assistance, and legal consulting. Except for huge conglomerates, most firms will not outsource work across all categories at the same time. The workload-generation process that we use has a single parameter p , which we call the *coherence parameter* of the workload, and works as follows. First, we start with a random task, which we select as pivot. To select the next task, with probability $1/p$ we select a random task from the pool of distinct tasks in the dataset and make this task the new pivot, and with probability $1 - 1/p$ we select another task with Jaccard similarity at least 0.5 to the pivot. The expected length of a sequence of “similar” tasks is p . Each workload stream that we create has 10K tasks. We also experimented with streams of up to 100K tasks, but we observed that 10K tasks suffice to expose the trends of the algorithms that we compare. We believe that in general a large value of p is realistic for a company, as customers would probably procure from it services exhibiting a certain coherence; we also evaluate our algorithms for a broad range of values for p .

For each dataset and for each coherence parameter that we use, we generated 100 workload streams; the costs that we report in our experiments are averages over these 100 workloads.

Cost parameters. We have data about the rates charged by workers in each marketplace, which we directly interpret as their outsourcing costs λ_r . However, we do not have their hiring or salary costs, so we experiment with different values for these costs.

For *hiring costs*, which are characterized by $C_r > \lambda_r$, we assume they are a multiplicative factor larger than the hiring cost, $C_r = \alpha_r \lambda_r$. We performed extensive experiments in which C_r varied between $1\lambda_r$ and $30\lambda_r$, either as a fixed value, or setting α_r to be a random variable distributed uniformly in a small range.

For *salary costs*, we assume that they are a fraction of outsourcing costs, experimenting with values from $\sigma_r = \lambda_r/100$ to $\sigma_r = \lambda_r/4$. Salaries σ_r are smaller than outsourcing costs λ_r because the latter includes many costs in which a company incurs when outsourcing [6], including: (i) outside-hired consultants are usually more highly paid per hour/day than regular employees for a company, (ii) there are transaction costs involved in locating and contracting and outsourced worker that do not exist for regular employees, and (iii) there are communication and management costs of handling someone external to a company.

5.2 Experiments with LUMP SUM

Baselines. We consider two baselines. The Always-Hire baseline solves the SETCOVER problem for finding a low-cost set of workers that cover the task’s uncovered skills and hires them. The Always-Outsource baseline never hires, instead it outsources to workers that cover the required skills for the task, by solving a SETCOVER problem instance.

Results. Figure 1 (Left) summarizes our results for LUMP SUM for workloads generated with the UpWork, Freelancer and Guru datasets, depicting total cost as a function of the number of tasks.

We observe that under all these workloads the algorithms behave similarly. Always-Outsource has cost proportional to the number

of tasks and is not competitive, its cost is mostly outside the range of Figure 1 (Left). As expected, Always-Hire performs the best in the long run, because if the number of tasks is large, hiring is a dominant strategy; however the online algorithm does not know the number of tasks. Experimentally, the LumpSum algorithm has a cost that does not exceed that of Always-Hire by more than a factor of 2, across all the scenarios that we tested. We note that for short sequences LumpSum has lower cost; this difference in the cost can sometimes be an order of magnitude smaller (plots omitted for brevity). We also note that although LumpSum-Heuristic can, theoretically, perform arbitrarily bad, in our experiments it performs quite well—although worse than the theoretically justified LumpSum.

Variations (plots omitted for brevity). Figure 1 (Left) is obtained with $C_r = 4\lambda_r$. We do not observe dramatic variations in the results when varying this parameter in the studied range ($1\lambda_r$ through $30\lambda_r$): LumpSum has a smaller cost than Always-Outsource. In general, higher hiring costs mean the number of tasks required before hiring a worker is larger, the costs of LumpSum and Always-Hire are higher, and the advantage for LumpSum over Always-Hire for a small number of tasks holds for a longer period of time.

In all plots of Figure 1 we use coherence parameter $p = 100$, which means we expect the input stream to be composed, on average, of runs of 100 similar tasks (i.e., having Jaccard coefficient of at most 0.5 between consecutive ones). In this setting, even if the workload is not coherent (experimentally, even for $p = 1$), LumpSum is still better than Always-Outsource.

5.3 Experiments with TFO

Baselines. As in LUMP SUM, we consider baselines Always-Hire and Always-Outsource. Additionally, we consider TFO-Heuristic (defined in Section 4.4), which does not have a theoretical guarantee.

Results. Figure 1 (Right) summarizes our results for TFO. We observe that TFO, TFO-Heuristic, and TFO-Adaptive have the smallest total cost, followed by Always-Outsource. In contrast, the Always-Hire strategy has much higher cost due to mounting salary costs. We also observe that while TFO-Heuristic does not offer the theoretical guarantees of TFO, it performs well in practice.

Variations. Similarly to LUMP SUM, varying C_r does not bring dramatic changes, but as C_r increases while maintaining workload coherence and salary to outsource cost ratios constant, the advantage of TFO over Always-Outsource decreases, and for large hiring costs Always-Outsource has the smallest cost (plots omitted for brevity). Concretely, for $p = 100$ and $\sigma_r = \lambda_r/10$, if we vary the hiring cost C_r (from $1\lambda_r$ to $30\lambda_r$), the total cost of TFO remains less or equal than the total cost of Always-Outsource until $C_r = 16\lambda_r$, when the cost of TFO becomes larger than the cost of Always-Outsource for the workload generated using the Guru dataset. The corresponding values of C_r for workloads generated with Freelancer and Upwork data are $C_r = 18\lambda_r$ and $C_r = 26\lambda_r$ respectively. As expected, if the hiring costs are sufficiently large, Always-Outsource becomes a dominant strategy.

Figure 2 (Left) compares TFO and Always-Outsource experimentally by varying the coherence parameter p from 20 to 200 and σ_r from $\lambda_r/50$ to $\lambda_r/4$. We observe that less coherent workloads and high salaries make hiring more expensive; Always-Outsource then becomes a dominant strategy. Figure 2 (Right) shows the power of

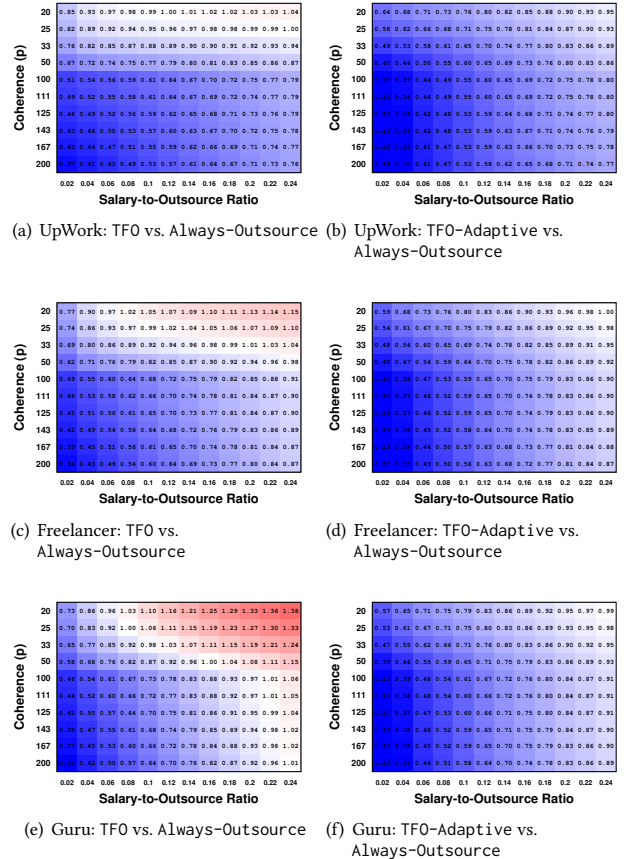


Figure 2: (Best seen in color.) Left: ratio of the cost achieved by TFO and Always-Outsource. Right: ratio of the cost achieved by TFO-Adaptive and Always-Outsource. Coherence parameter p varies from 0.02 to 0.24; salary-to-outsource ratio varies from 20 to 200; the number of tasks is 10K. Colors represent the ratios of costs: blue (dominant towards the bottom-left) indicates the region where our algorithms TFO and TFO-Adaptive have smaller cost, while red indicates the region where the baseline Always-Outsource has smaller cost. In the white region both algorithms have similar costs.

the TFO-Adaptive algorithm. We observe that it performs equal or better than Always-Outsource for all the range of parameters.

Performance. Our code, which will be released with this paper, is a relatively straightforward mapping of the algorithm to simple counters. Written in Java, it requires about 5 to 8 seconds on average to process 10K incoming tasks using commodity hardware. We remark that, although our formulation is a linear program, the method does not involve *solving* the linear program, instead, we obtain the solution using the specific primal-dual method that we have described and analyzed.

6 RELATED WORK

To the best of our knowledge, we are the first to introduce and solve the Team Formation with Outsourcing (TFO) problem. However, our work is related to existing work on crowdsourcing, team formation, and online algorithms design, which we outline next.

Crowdsourcing. Among the extensive literature in crowdsourcing, the most related to ours is the work of Ho and Vaughan [13]. Their goal is to assign individual workers to tasks, based on the workers' skills. Although Ho and Vaughan also deploy the primal-dual technique to solve the task-assignment problem, the tasks they consider can be performed by individual workers and not by teams. Thus, both their problem and their algorithm is different from ours.

Team formation. A large body of work in team formation considers the following problem: given a social or a collaboration network among the workers and a set of skills that needed to be covered, select a team of experts that can collectively cover all the required skills, while minimizing the communication cost between the team members [2, 4, 10, 11, 16, 18, 19, 27]. Other variants of this problem have also considered optimizing the cost of recruiting promising candidates for a set of pre-defined tasks in an offline fashion [12] and minimizing the workload assigned to each individual team member [3, 21].

Although the concept of set-cover is common between our work and previous work, the framework we propose on this paper is different in multiple dimensions. First, we do not focus on optimizing the communication cost; in fact we do not assume any network among the individual workers. Our goal is to minimize the overall cost paid on hiring, outsourcing, and salary costs. This difference in the objectives leads to different (and new) optimization problems that we need to solve. Secondly, most of the work above focuses on the offline version of the team-formation problem, where the tasks to be completed are a-priori known to the algorithm. The exception is the work of Anagnostopoulos et al. [3, 4]. However, in their setting they aim to distribute the workload as evenly as possible among the workers, while our objective is to minimize the overall cost of maintaining a team that can complete the arriving tasks. Moreover, the option of outsourcing that we propose is new with respect to the team formation literature. Finally, in the design of our online algorithms we use the primal-dual framework, which was not the case for previous work on online team formation.

Primal-dual algorithms for online problems. The algorithms we design for our problems use the primal-dual technique. A thorough analysis on the applicability of this technique for online problems can be found in the book by Buchbinder and Naor [7] and in [5]. Probably the most closely related problem are the *ski-rental* and the *set cover* problems. We have already discussed the connection of TFO to ski-rental and set cover in Section 2. One can also draw the analogy with caching; one can think that bringing a page to the main memory is analogous to hiring a person. The main differences are that in the typical caching problem we do not have covering constraints, there are no recurring costs for keeping pages in the cache, and there is a fixed limit on the number of pages we can insert in the cache.

7 CONCLUSIONS

In this paper, we introduced and studied *Team Formation with Outsourcing*. We showed that hiring, firing, and outsourcing decisions can be taken by an online algorithm leading to cost savings with respect to alternatives. These cost savings are more striking when (1) the hiring and salary costs are low, because then hiring becomes an attractive option; (2) the tasks exhibit high coherence, i.e., consecutive tasks are similar to each other; and (3) the time horizon is long enough that we can find a core pool of workers to stay hired and satisfy a large fraction of the skills required by incoming tasks.

Technically, the problems we have analyzed in this paper involve embedding a set-cover problem in an online algorithm. Our main algorithms (LumpSum, TFO) are able to give results that are competitive in practice and, equally importantly, theoretically close to the best one can hope for. The design of our algorithms is based on the online primal-dual technique; we provide an experimental evidence of the goodness of this method even for a complex real-world problem. Furthermore, we present two heuristics which, although in theory are not competitive, perform well in practice. Future work may extend this by considering worker compatibility [4, 18], learning of new skills by hired workers, or other extensions.

Future work. As most problems, we can introduce further elements to introduce even more generality. For instance, the algorithms we have described assume one and only one task arrives per unit of time, can be extended trivially to cases in which task arrivals occur at arbitrary times.

As we noted in Section 6, there are also parallels with scenarios of caching and paging. Extending TFO when the number of hired workers is limited turned out to be a challenging combination of set cover, weighted caching and ski rental. We have begun to study these problems, our preliminary results show that we can achieve a $O(\log k \log m)$ approximation, in which k is the maximum size of the worker pool. A more natural constraint could be that, for instance, the total cost paid per unit of time cannot exceed a certain budget, which would represent a cap in weekly or monthly personnel expenses. Another element we could incorporate is the possibility of not handling a task, but instead paying a penalty when a task is too difficult to handle with current workers and it is expensive to replace the worker pool with new workers. Other variants can include workers with different ability levels. We plan to study some of these variants in future work.

Additionally, we note that all the algorithms we have presented in this paper are deterministic. Just as randomized algorithms for paging can be defined in the primal-dual framework [7], it is of interest to introduce other update rules for the primal variables that allow us to describe a randomized algorithm.

Reproducibility. The code and data of this paper can be found at <https://github.com/adrianfaz/Algorithms-for-Hiring-and-Outsourcing-in-the-Online-Labor-Market>.

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